

Testing Functions of One and Two Arguments

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Abstract

Software to evaluate functions of one or two arguments is too often tested or certified, or the accuracy assessed, by *ad hoc* methods.

Systematic methods to test, certify or assess the accuracy of single- or double-precision functions of one or two arguments, and supporting software, are described here.

1. introduction

Testing the accuracy of procedures that evaluate functions of one or two arguments has taken many forms, from code inspection, to comparing a few selected values produced by the procedure with tabulated values, to cumbersome testing protocols inspired by methods of software engineering, that verify that every line of code of the procedure is executed.

The first of these methods addresses the question whether the procedure attempts to solve the desired problem. The second addresses the question whether the procedure calculates the correct answer. It addresses the question of real interest, but because of the manual nature of the process it is tedious and inherently unreliable. The third addresses the question whether the input has been sufficient to exercise all of the procedure, but not the question whether the procedure computes the correct result.

We describe here a simple yet comprehensive testing protocol, of the second kind described above, together with supporting software that automates the process, thereby removing the tedium, and making the process more reliable.

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In outline, the method consists of

- Executing the software under test with systematically chosen arguments in user specified ranges;
- Comparing each result produced by the software under test to a result computed by a higher-precision reference procedure, and thereby calculating four error measures, *viz.*

Error in units of the least significant digit,

Relative error in units of the machine precision,

- Absolute error, and

Error relative to the inherent error of the function;

- Accumulating statistics and a histogram for each error measure; and

- Reporting the test results, *viz.*

The statistics and histograms;

The ten most positive and ten most negative errors, by each measure; and

A crude ‘{plot’) of the errors, by each measure.

The supporting software consists of two single-precision procedures, STFN1 to test functions having one argument, and STFN2 to test functions having two arguments, and two corresponding double-precision procedures, DTFN1 and DTFN2. If *clow*, we use xTFN1 or xTFN2 where the discussion applies to both precisions. xTFN1 and xTFN2 use a common 10W-1CVC1 procedure, xTFN, and others not discussed here.

2 Selecting argument values

2.1 Selecting argument values in xTFN2

The calling procedure optionally sets parameters NX, NY, MX and MY. NX and NY specify the number of major subdivisions for the first and second arguments, respectively; MX and MY specify the number of minor subdivisions for the first and second arguments, respectively; The default values for NX, NY, MX and MY are 72, 36, 4 and 8, respectively.

The calling procedure provides the argument array RANGES(1:4) to xTFN2.

xTFN2 divides the rectangular region RANGE(1:2) x RANGE(3:4) into NX x NY major regions, and each of those into MX x MY minor regions. Then one pair of argument values is selected in each minor region from two independent uniform random distributions.

2.2 Selecting argument values in xTFN1

The calling procedure provides arguments RANGE(1:2) and NPTS to xTFN1 to specify the range and number of arguments at which to test the function.

xTFN1 sets NX = 25, MX = [NPTS / 25], NY = 1 and MY = 1, and calls xTFN.

If one wants different values for NX and MX, one can use xTFN2 with NY = 1 and MY = 1.

3 Measuring the errors

Each time xTFN requests a function value, the user's function evaluation procedure is expected to provide a result from the function procedure under test, and a reference result of higher precision, which is assumed to be accurate to the full precision of the test result. When testing a single-precision function, the user is expected to provide a double-precision reference result. When testing a double-precision function, the user is expected to provide an extended precision reference result in a format, compatible to Richard Brent's extended precision package [1, 2], which is used internally by DTFN.

Below, b is the radix of arithmetic, the test value is $V_t = F_t b^{E_t}$, the reference value is $V_r = F_r b^{E_r}$, ν is the number of base b digits in arithmetic used by the procedure under test, and $\epsilon \approx b^{\ell}$ is the machine precision, the smallest number such that $1.0 + \epsilon \neq 1.0$ when evaluated in the arithmetic of the procedure under test. xTFN assumes floating-point numbers are normalized.

Errors are measured in four ways:

Error as a multiple of the least significant digit is $E = W_t - W_r$, where $W_t = F_t b^\nu \approx V_t b^{\nu - E_t}$ and $W_r = V_r b^{\nu - E_t} = F_r b^{\nu + E_r - \nu}$.

Absolute error as a multiple of the working precision ϵ is $A = Eb^{\nu_t - 1} = D/\epsilon$, where $D = V_t - V_r$. Scaling by ϵ is not always ideal, but is usually convenient for printing.

Relative error as a multiple of the working precision ϵ is $R = \frac{E}{|W_t|}$, $b^{\nu_t - 1} = \frac{D}{|V_r|} b^{\nu_t - 1} = \frac{A}{|V_r|}$. The ratio E/R is bounded between $1/b$ and b .

Error compared to the inherent error is

$$B = \frac{D}{\epsilon \left(\sum_{i=1}^{\nu} \left(\left| x \frac{\partial V_{t,i}}{\partial x} \right| + \left| y \frac{\partial V_{t,i}}{\partial y} \right| \right)^2 \right)^{1/2}},$$

in which the denominator is how one expects an error in the arguments of one

Unit of round-off to affect the function value; nv is the number of components of the function value and $V_{t,i}$ is the i 'th component of the test value. An error $B = 1$ is in some sense “the Best one can expect.”

The user's function evaluation procedure can compute derivatives, or request that xTFN approximate derivatives by differences. In the latter case, the function evaluation procedure can signal that the function is an analytic function of a complex argument, in which case $\frac{\partial V_{t,1}}{\partial y}$ and $\frac{\partial V_{t,2}}{\partial y}$ can be computed from $\frac{\partial V_{t,1}}{\partial x}$ and $\frac{\partial V_{t,2}}{\partial x}$ by applying the Cauchy-Riemann conditions.

xTFN can assess functions that return results with more than one component, in particular, complex results. If a result has one component, errors are measured as explained above. If a result has more than one component, the error in the least significant digit is computed as above but only for the component of largest magnitude; absolute error $D = \|V_t - V_r\|_2$; scaled absolute error $A = D/\epsilon$ and relative error $R = A/\|V_r\|_2$.

4 Statistics and histograms

4.1 Statistics

For each measure, xTFN computes mean, standard deviation, minimum and maximum error. Statistics are separately computed for signed and unsigned error measures.

For each measure, xTFN remembers the arguments and function values for which the 10 most positive and 10 most negative errors were observed.

4.2 Histograms

xTFN computes histograms of the number of samples for which $E < -2^{10} \leq E < -2^9 \leq \dots < -1/2 \leq E < 0 \leq E \leq 1/2 < E \leq 1 < \dots \leq 2^9 < E \leq 2^{10} < E$. A similar histogram (with only positive ranges) is computed for $|E|$. Similar histograms are computed for the other error measures R , A and B .

5 Reports

Results of the tinting are presented in three parts.

The first part presents the argument ranges, number of samples attempted and number of samples successfully evaluated, histograms, and statistics of mean, standard deviation, minimum and maximum.

The second part presents the 10 most positive and 10 most negative errors observed, for each measure, together with the arguments and function values for which those errors were observed.

The third part consists of a crude "plot" of the errors, as described in the next section.

Each page of the report includes a user-sujqlicxl title.

6 Crude "plots" of the errors

In each of the NX x NY major subdivisions of the arguments, xTFN remembers the minimum and maximum error encountered in any of the MX x MY minor subdivisions, for each measure. xTFN produces crude plots of the errors using these minimum and maximum error summaries. The user can select the error measures for which plots are constructed.

For functions of one argument, a crude plot of the maximum and minimum error in a major subdivision is prepared by constructing a character array in which a symbol ('E', 'R', 'A' or 'B') is placed at a position in the array corresponding to the value of the error for that subdivision]], scaled to the minimum and maximum error for the entire range tested. If zero is within the range of minimum and maximum error for the entire range tested, a '0' symbol is placed at the appropriate position before 'E', '1{', 'A' or 'B' is placed into the array.

For each major subdivision, the abscissa of minimum error, the minimum error, and the plot of minimum error is printed, for all selected error measures, on the same line, followed by another line displaying similar information for maximum error.

For functions of two arguments, a crude "density plot" of the errors is constructed. For each of the NY major subdivisions of the second argument, a character array of NX elements is constructed, one for each of the NX major subdivisions of the first argument. A symbol is placed into each element of the array, that depends on the error of maximum magnitude, and the sign of the error of maximum magnitude. The default, values are blank for no data, '.' for $0 \leq |e| \leq 1/2$, 'A' for $1/2 < e \leq 1$, 'B' for $1 < e \leq 2$, etc., and lower case letters for negative errors (e is one of E, R, A or B). The user can change the symbols.

The plot is constructed, row by row, by printing the Y abscissa and a character array constructed as described above, for each of the NY major subdivisions of the second argument. A separate page is used for each selected plot.

Acknowledgements

The procedures xTFN are based on earlier procedures of the same names [4], and programs xT1 and xT4 [3], developed by Charles L. Lawson to test functions of one argument.

Most of the important ideas in the present version of xTFN are directly derived from the earlier work by Lawson. "Inherent error" derives from work by Lawson for which no formal report was prepared. Lawson discussed results of a program RFAC, that illustrated "inherent error," informally at an IFIP TC2 WG 2.5 meeting in late 1988.

References

- [1] Richard P. Brent, *A Fortran multiple-precision arithmetic package*, **ACM Transactions on Mathematical Software** **4**, 1 (March 1978) 57-70.
- [2] Richard P. Brent, *An algorithm 524: MP, A Fortran multiple-precision arithmetic package [A 1]*, **ACM Transactions on Mathematical Software** **4**, 1 (March 1978) 71-81.
- [3] Charles L. Lawson, **Methods of Function Subprogram Testing Implemented in the Subroutine ST4**. Internal Computing Memorandum 505, Jet Propulsion Laboratory, (March 1984).
- [4] Charles L. Lawson, **STFN and DTFN - Subroutines for Testing Fortran Function Subprograms**. Internal Computing Memorandum 528, Jet Propulsion Laboratory, (July 1988).

A Sample output from STFN2

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ClassifyErrors in SinglePrecision Subroutines          03/26/96
Reference: ZWOFZ - Double precision complex W
Test: CWOF2 - Singleprecisioncomplex W

First argument: ReferenceRange = [ .0000 00E+ DO. 10.0000   1, divided into 288 equal subranges
Second argument: ReferenceRange = [ .0000 00E+DO. 10.0000   1, divided into 288 equal subranges,
with a point randomly selected in each of the 82944 subranges.
Reference and test functions were successfully executed, and the results successfully analyzed, at 81627 points

EPS = 1.1921E-07
D = Ytest - Ytrue
E = D in units of last bit position of Ytest      A = D / EPS
R = REI/EPS                                         REI = D ; ABS(Ytrue)
                                                 B = D / max(EPS*ABS(Ytrue),ABS(PerturbedYtrue-Ytrue))

GOOD    CLASSIFICATION      E          R          A          B
BITS     INTERVALS        COUNT %  COUNT %  COUNT %  COUNT %  COUNT %  COUNT %
                                                 COUNT %  COUNT %  COUNT %
                                                 COUNT %  COUNT %

      BELOW -2**10   2   .0      2   .0      0   .0      1   .0
13 -2**9 To -2**10  1   .0      1   .0      0   .0      1   .0
14 -2**8 To -2**9  3   .0      0   .0      0   .0      1   .0
16 -2**7 To -2**8  4   .0      6   .0      0   .0      0   .0
16 -2**6 To -2**7 13   .0      2   .0      0   .0      3   .0
17 -2**5 To -2**6 24   .0      20   .0      0   .0      6   .0
18 -2**4 To -2**5 66   .1      37   .0      0   .0      13   .0
19 -2**3 To -2**4 474   .6      152   .2      0   .0      36   .0
20 -2**2 To -2**3 1361  1.7      937   .1      0   .0      318   .4
21 -2**1 To -2**2 1963  2.4      1806  2.2      21   .0      1168  1.4
22 -1.0 To -2**1 2920  3.6      2163  2.6      197   .2      1960  2.4
23 -0.6 To -1.0 4643  6.7      3'300  4.8      630   .6      2731  9.3

24 -0.0 To -0. s 9892 12.1      12341 16.1      20618 26.3      16148 18.6
24 0.0 To 0.5 16616 18.1      22560 27.6      68128 72.4      32934 40.3

23 0.6 To 1.0 16602 20.3      20719 25.4      844   1.0      17744 21.7
22 1.0 To 2**1 18641 22.8      12114 14.8      266   .3      6994  8.6
21 2**1 To 2**2 6268 7.7      3144  3.9      24   .0      1987  2.4
20 2**2 To 2**3 2264 2.8      1369  1.7      0   .0      520   .6
19 2**3 To 2**4 71a  .9      261   .3      0   .0      69   .1
18 2**4 To 2**5 124   .2      61   .1      0   .0      12   .0
17 2**5 To 2**6 29   .0      14   .0      0   .0      6   .0
16 2**6 To 2**7 10   .0      6   .0      0   .0      1   .0
16 2**7 To 2**8 4   .0      3   .0      0   .0      2   .0
14 2**8 To 2**9 2   .0      2   .0      0   .0      1   .0
13 2**9 To 2**10 2   .0      1   .0      0   .0      2   .0
ABOVE 2**10 2   .0      2   .0      0   .0      0   .0

|E| CUMULATIVE |R| CUMULATIVE |A| CUMULATIVE |B| Cumulative
24 0.0 To 0.5 2550731.2 2550731.2 34901 42.0 34901 42.8 79746 91.1 79746 97.1 48082 68.9 48082 68.9

23 0.6 To 1.0 21245 26.0 46762 57, S 24619 30, > 69520 72.9 1374 1.7 81120 99.4 20476 26.1 68557 84.0
22 1.0 To 2**1 21661 26.4 68313 83.7 14277 17.6 73797 90.4 462 ,68158299.9 8'344 11.0 77601 94.0
21 2**1 To 2**2 6221 10.1 76634 93.8 4960 6.1 78747 96.6 45 .1 81627 100.0 3145 3.9 80646 95.8
20 2**2 To 2**3 3615 4.480149 98.2 2306 2.8 81063 99.3 0.0 81627 100.0 838 1.0 81484 99.8
19 2**3 To 2**4 1192 1.S 81341 99.6 419 .5 81472 99.8 0 81627 100.0 95 .1 81579 99.9
18 2**4 To 2**5 190 .2 81631 99.9 98.1 81670 99.9 0 .0 81627 100.0 25 .0 81604 100.0
17 2**5 To 2**6 63 .1 81584 99.9 34 .0 81604 100.0 0 .0 81627 100.0 11 .0 81616 100.0
16 2**6 To 2**7 23 .0 81607 100.0 1 .0 81611 100.0 0 .0 81627 100.0 4 .0 81619 100.0
16 2**7 To 2**8 8 .0 81615 100.0 8 .0 81619 100.0 0 .0 81627 100.0 2 .0 81621 100.0
14 2**8 To 2**9 5 .0 81820 100.0 2 .0 81621 100.0 0 .0 81627 100.0 2 .0 81623 100.0
13 2**9 To 2**10 3 .0 81623 100.0 2 .0 81623 100.0 0 .0 81627 100.0 3 .0 81626 100.0
ABOVE 2**10 4 .0 81621 100.0 4 .0 81627 100.0 0 .0 81627 100.0 1 .0 81627 100.0

E |E| R IRI A IAI B IB
MEAN .612 1.634 .391 1.148 2.487 E-02 6.608 E-02 .292 .122
ST. DEV. 28.569 28. S28 23.142 23.117 .168 .161 11.308 11.288
MAX POS. VALUE 203 S.983 7116.639 1649.919 6039.006 3.06 3.80 896.261 2826.624
MAX NEG. VALUE -1116.639 -6038.006 -3.00 ->826.624

REI |REI| D IDI
MEAN 4.669 E-0 8 1.368E-07 2.965E-09 7.878E-09
STD. DEV. 2.169 E-06 2.766 E-06 2.007E-08 1.869E-08
MAX POS. VALUE 1.848 E-04 7.199 E-04 3.640 E-07 4.529E-07
MAX NEG. VALUE -7.199E-04 -4.529E-07

```

To save space in this report, output of the ten most positive and negative errors is provided here only for the relative error measure, R . Floating-point numbers described as "Hex" are displayed with hexadecimal fraction. The exponent is to the base b used in the arithmetic of the procedure under test, which may not be hexadecimal. The exponent is printed in decimal notation. For example, using IEEE arithmetic, the decimal number 0.62437×10^{-3} is printed in "Hex" as "-10 +A3AC8F" which means $0.A3AC8F_{16} \times 2^{10}$, where "10" is the decimal number **10**, not **2** (binary 1 0) or 16 (hexadecimal 10).

Classify Errors in Single Precision Subroutines

03/25/96

Reference: ZWDFZ - Double precision complex W
Test: CWDFZ - Single precision complex W

The 10 most Positive values of R

Part	Decimal Argument	Hex Arg.	Hex Test	Hex Ref	Decimal Error
	Exp	Frac	Exp	Frac	
1	4.9553	3 +9E9172	-15 +S09998	-1b +809383 1	1550.
2	.62437E-03	-10 +A3AC8F			
1	4.9228	3 +9D87DE	-14 +D7B25E	-14 +D7AA6A 0	1208.
2	20653E-02	-8 +87594F			
1	6.6666	3 +B554EF	-16 +8E3184	-16 +8E2F46 D	516.0
2	.45902E-03	-11 +FOA856			
1	3.3258	2 +D4D9BB	-11 +DC0E55	-1 1 +DC0C26 E	324.6
2	.66838E-02	-7 +DB03F4			
1	4.7160	3 +96E913	-12 +A4FA1B	-12 +A4F8AE D	282. b
2	57609E-02	-7 +BCC5C4			
1	2.92B9	2 +BB7341	-11 +9EE733	-11 +9E.F60C 9	237.1
2	.13730E-02	-9 +B3F6B2			
1	6.9494	3 +BE61DF	-13 +839733	-13 +83965F S	205.7
2	37644E-02	-8 +F6B4E1			
1	3.9216	2 +FAFA0B	-11 +92E57F	-11 +92E4FB 2	126.8
2	68209E-02	-7 +DF819B			
1	3.6867	2 +EBF3AF	-10 +B4D259	-10 +B4D1BE b	109. b
2	14561E-01	-6 +EE9280			
1	3.8607	2 +F715BC	-11 +BFF8DD	-11 +BFF83F 8	105.0
2	.86015E-02	-6 +8CED35			

The 10 most Negative values of R

Part	Decimal Argument	Hex Arg.	Hex Test	Hex Ref	Decimal Error
	Exp	Frac	Exp	Frac	
1	4.2468	3 +87E5EF	-16 +96B58B	-16 +96D156 8	-6039.
2	.26134E-03	-11 +8904AB			
1	5.2744	3 +A72E46	-1b +ED2BD4	-1b +ED3623 B	-1424.
2	.12896E-02	-9 +A90979			
1	4.7567	3 +9836ED	-13 +9C1CEF	-13 +9C1F75 7	-630.0
2	.27771E-02	-8 +B60004			
1	3.2919	2 +D2AEE4	-11 +994642	-11 +99474C o	-222.2
2	.43992E-02	-7 +902756			
1	5.6105	3 +B05621	-15 +B4664E	-15 +B4675C 2	-191.7
2	.10982E-02	-9 +8FF032			
1	5.2846	3 +A91B2F	-13 +B46F55	-13 +B4704E E	-177.3
2	40207E-02	-7 +B3BFBF			
1	4.0592	3 +B1F49C	-11 +B49764	-11 +B49855 D	-171.4
2	90685E-02	-6 +94941D			
1	3.3734	2 +D7E62A	-12 +FC1644	-12 +FC1773 8	-154.1
2	39224E-02	-7 +808756			
1	3.6460	2 +E958CC	-10 +CB2ER6	-10 +CB2F71 D	-88.08
2	.15939E-01	-5 +829389			
1	4.7716	3 +98B15B	-11 +FB8C52	-11 +FB8CFC F	-87.00
2	18020E-01	-5 +939F1B			

Classify Errors in Single Precision Subroutines

03/25/96

Reference: ZWOFZ - Double precision complex W
 Test: CWOFZ - Single precision complex W

"Contour" plot of R.

Positive errors with O..26 incorrect bits are represented by .ABCDEFHIJKLMNOPQRSTUVWXYZ
 Negative errors with O..25 incorrect bits are represented by .abcdefghijklmnoprstuvwxyz

9.72222	BB
9.44444	BBCBH
9.16667	BBBBBBBBBBBBBBABBBBABA BBBCBBBBBBBBBBBBCB
8.88889	BBBBBAAAAAABBcBBBBBBBB
8.61111	BcBBBcBcBBBBBBBB
8.33333	BB
8.05556	BBCScBCBBBBBBBB
7.77778	BBcBBBBBBBBBBBBcB8
7.50000	BBBBBCCCCBB
7.22222	BBCB
6.94444	BBCBBCCBCBBBBBBBB
6.66667	BBCBH
6.38889	BBBcBBBBBBBB
6.11111	BBBBBBBBBB BBBB
6.63333	BBAcBBBBBBBBBAABB
6.55556	BABBcBBBB
6.27778	BBBcBBBB
5.00000	BBCCBCBBBBCCBB
4.72222	BABBBCCBBB
4.44444	BBBBBABBBCCBCBBBBCCBCBBBBCCBB
4.16667	CCCCC CCC
3.88889	DCC
3.61111	DccCDDdCCCCCCCCCCCCc CCC
3.33333	CCCCcCDCCCCDCCcCDCCCCDCCCDCCcCBBBBBCCBVA CBBCCBCCBCCBCCBCCBCCBCCBCCBCC
3.05556	cCcDdDcDcDcDcDcDdDc
2.77778	ddDDdDEdDDDCDD
2.60000	ddDDDCdDDDDd
2.22222	Dd
1.94444	dDd
1.66667	deeEFDeDd
1.38889	dEeDd
1.11111	eeDd
.833333	bccCCdDEEDDEDDDEdEdeeEEEdedeEeEdedDDDDdDdDdDdDdDdDdDdDdDdDdDdDdDdDdDd
.655556	BCDEEEeEEeEEeEEDEDEdEeDEEEFEdDd
.277778	bdeEfEEf
.000000E+00	DEEFGgGGgIgJiGHHlIngHjkLgliikgIHFeBBBBChcBBcBBBBCCBCBCCCCCBB

B Sample output from STFN1

The first page of output from STFN1 is similar in format to output from STFN2. In the second part of the output, printing the ten most positive and negative errors, STFN2 must print two argument values, while STFN1 prints only one. To save space in this report, these outputs from STFN1, which are similar to the first two pages of output shown above for STFN2, are not shown here. The plot produced by STFN1 is substantially different from the plot produced by STFN2, as shown below. In this output, the user has selected plots only for error measures E , R and B .

Classify Errors in Single Precision Subroutines							03/19/96			
Reference: DSIN			Double precision SINE							
Test: SIN			Single precision SINE							
6.5976E-02	-.48 E	0	2.0948 E-02	-.33	R	0	6.6976 E-02	-.01	B	0
2.8602E-02	.60	0	E 3.1629E-02	.47	0	0	R 3.1629E-02	.01	0	0
8.2260 E-01	-.49E	0	6.5683E-02	-.40	R	0	B.2260 E-02	-.03	B	0
8.2304 E-02	.60	0	E 8.2304E-02	.38	0	0	R 8.2304E-02	.03	0	0
.1813	-.49E	0	.1386	-.37	R	0	.1813	-.06	B	0
.1494	.60	0	E .1291	.48	0	0	R .1494	.06	0	0
.2268	-.47 E	0	.2268	-.26	R	0	.2268	-.06	B	0
.2105	.46	0	E .1962	.28	0	R	.2106	.06	0	B
.2639	-.48E	0	.2639	-.46	R	0	.2639	-.12	B	0
.3009	.49	0	E .2533	.48	0	R	.3009	.12	0	B
.3440	-.49E	0	.3376	-.37	R	0	.3440	-.12	B	0
.9328	.50	0	E .3326	.38	0	R	.3328	.12	0	B
4.096	-.50E	0	.3836	-.33	R	0	.4096	-.12	B	0
.3929	.49	0	E .3929	.32	0	R	.3028	.12	0	B
.5015	-.50E	0	.5016	-.26	R	0	.5016	-.12	B	0
.4794	.49	0	E .4794	.26	0	R	.4794	.12	0	B
.6498	-.49E	0	.6498	-.47	R	0	.6498	-.26	B	0
.6234	.60	0	E .6747	.60	0	R	.5247	.26	0	B
.6832	-.50E	0	.5832	-.45	R	0	.5832	-.26	B	0
.6070	.50	0	E .6070	.44	0	R	.8070	.26	0	B
.6872	-.49E	0	.6371	-.40	R	0	.6872	-.24	B	0
.6463	.49	0	E .6463	.41	0	R	.6463	.24	0	B
.7213	-.50E	0	.7213	-.38	R	0	.7213	-.26	B	0
.7230	.60	0	E .6924	.39	0	R	.7230	.25	0	B
.7993	-.49E	0	.7664	-.36	R	0	.7993	-.24	B	0
.7627	.49	0	E .7627	.36	0	R	.7621	.24	0	B
.8748	-.49E	0	.8408	-.83	R	0	.8748	-.26	B	0
.8693	.50	0	E .8211	.33	0	R	.8593	.25	0	B
.9216	-.48 E	0	.0027	-.29	R	0	.9216	-.22	B	0
.9132	.49	0	E .9132	.31	0	R	.9132	.24	0	B
.9938	-.49E	0	.9555	-.30	R	0	.9938	-.26	B	0
.9949	.49	0	E .9437	.30	0	R	.9949	.26	0	B
1.021	-.50E	0	1.021	-.29	R	0	1.021	-.26	B	0
1.017	.47	0	E 1.017	.28	0	R	1.017	.24	0	B
1.074	-.50E	0	1.014	-.28	R	0	1.074	-.26	B	0
1.107	.49	0	E 1.078	.28	0	R	1.107	.24	0	B
1.185	-.50E	0	1.185	-.27	R	0	1.185	-.26	B	0
1.176	.60	0	E 1.141	.27	0	R	1.176	.25	0	B
1.241	-.49E	0	1.247	-.26	R	0	1.241	-.26	B	0
1.703	.60	0	E 1.203	.27	0	R	1.203	.26	0	B
1.303	-.49E	0	1.303	-.26	R	0	1.303	-.24	B	0
1.302	.60	0	E 1.302	.26	0	R	1.302	.26	0	B
1.312	-.48 E	0	1.372	-.24	R	0	1.372	-.24	B	0
1.348	.60	0	E 1.348	.20	0	R	1.348	.25	0	B
1.388	-.50E	0	1.386	-.25	R	0	1.386	-.26	B	0
1.440	.41	0	E 1.440	.24	0	R	1.440	.24	0	B
1.499	-.48 E	0	1.499	-.24	R	0	1.499	-.24	B	0
1.496	.60	0	E 1.476	.26	0	R	1.496	.25	0	B
1.835	-.50E	0	1.635	-.26	R	0	1.635	-.26	B	0
1.562	.60	0	E 1.562	.25	0	R	1.552	.26	0	B